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Marketable Permits vs. Emission Fees: Uncertain Costs, Damage and Product Demand

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Abstract

In this paper we extend Weitzman's result on the relative desirability of price controls versus quantity controls. We consider the case where pollution is produced jointly with a marketed good and the regulator much choose between marketable permits and emission fees for controlling pollution. In a climate of uncertainty, we show the conditions under which one regulatory instrument is preferred to the other and show that uncertainty in either good demand or costs, or both, is sufficient to yield a preference for one instrument over another. Uncertainty in damages is irrelevant.

I. INTRODUCTION

In a seminal paper a decade ago, Martin Weitzman (1974) considered the effect of uncertainty on the choice between price controls and quantity controls for regulating a firm's output of a good. His basic conclusion was that the existence of production cost uncertainty rather than demand uncertainty governed whether there would be a difference between these two forms of control. With such uncertainty, price controls are preferred if and only if costs have more curvature than benefits; quantity controls are preferred if and only if benefits have more curvature than costs. Apparently independently, Adar and Griffin (1976) and Yohe (1976) developed similar results.

The purpose of this paper is to extend Weitzman's result to the case where pollution is produced jointly with a good and it is the pollution which is to be regulated. In this context uncertainty can exist in any or all three of good demand, pollution/good production costs or pollution damage. The question we explore is what types of uncertainty are important to the relative desirability of emission fees vs. marketable permits; and further, what characteristics of costs, demands and damage influence the preference of one mechanism over another.

Our conclusion is that good demand and/or cost uncertainty is necessary for there to be a preference of one instrument over another. And, the preference depends on the slope of pollution damage and good demand as well as own- and cross-elasticities of marginal cost with respect to goods and pollution output.

II. THE MODEL

As does Weitzman (1974), we consider a rather simplified problem. We consider a competitive industry consisting of many price-taking firms. For the industry as a whole, amount g of a certain commodity is produced jointly with pollution b at industry cost C(g,b). Pollution yields damage D(b) and goods yield benefits B(g) equal to the consumer surplus associated with the inverse demand function for the good P(g). Thus,

$$B'(g) = P(g) \tag{1}$$

We assume industry costs are strictly convex and that demand is not upward sloping.

The regulator's problem is to maximize total surplus

$$S = B(g) - C(g,b) - D(b).$$

But the regulator is not completely free to adjust S. We assume he only controls b, and even that incompletely. In fact, we will limit regulation to be either a marketable permit issuance of $\bar{b}$ or an emission fee $\tilde{p}$. In the case of marketable permits, firms are allowed to buy and sell emission permits so that the allowed aggregate emissions, $\bar{b}$, will be achieved in some efficient manner (Montgomery, 1972). If the regulator announces an emission fee, all firms pay an amount equal to the product of the fee rate, $\tilde{p}$, and their emissions.

Complicating the regulator's decision is the presence of uncertainty as to what market demand, industry cost and pollution damage really are. The regulator has a perception of these functions, but clouded by uncertainty. This presents no difficulty if the regulator can continually adjust the number of outstanding pollution permits or the level of the emission fee. Generally, however, this is impractical. Thus we take the simplified view that the regulator must decide once and for all, on the optimal level of permits, b^*, or emission fee, p^*, based on *ex ante* perceptions and then live with whatever inefficiencies are revealed after industry and the environment respond to b^* and p^*.

To introduce uncertainty, we let good demand, costs and damage be parameterized by three independently distributed random variables, η, θ, and ε. These variables reflect the true state of the world which eventually (too late) is revealed to the regulator.

Surplus is then given by

$$S(g,b,\eta,\theta,\varepsilon) = B(g,\eta) - C(g,b,\theta) - D(b,\varepsilon). \qquad (2)$$

We assume S is strictly concave with respect to g and b. Convexity of D(b) is sufficient for this, but overly restrictive. The regulator will choose b^* or p^* to maximize the expected value of S.

It should be pointed out that while the regulator does not know η, θ, or ε, the industry being regulated does know η and θ either explicitly or can rapidly learn their values by rapidly adjusting output in response to market forces.

We now consider how firms will respond to a permit issuance and an emission fee.

A. <u>Industry Response to Regulation</u>

Our concern here is how industry reacts to a paticular permit issuance $\overline{b}$ or emission fee $\tilde{p}$. Our goal is to develop reaction functions $g(\overline{b})$, $b(\overline{b})$, $g(\tilde{p})$ and $b(\tilde{p})$ which can be substituted into equation (2). Surplus will then only be a function of random variables and the level of the regulation so the optimal expected-surplus-maximizing regulation can be found.

Consider first the permit issuance. If the regulator releases $\overline{b}$ permits to pollute, firms will respond by providing $\overline{b}$ pollution in the aggregate. Production of goods will be determined by the market:

$$P(g,\eta) = C_1(g,\overline{b},\theta) \tag{3}$$

where C_1 denotes the partial derivative of C with respect to the first argument. Thus, the response $\overline{g}$ to any (optimal or non-optimal) permit issuance $\overline{b}$ is the solution of equation (3).

Now consider the imposition of an emission fee, $\tilde{p}$. Industry will choose output levels $\tilde{g}$ and $\tilde{b}$ to maximize profits. In particular, marginal costs will be set equal to the fee for pollution and to good price for the good. For any fee, $\tilde{p}$, the resulting $\tilde{g}$ and $\tilde{b}$ will satisfy

$$P(g,\eta) = C_1(g,b,\theta) \tag{4a}$$

$$\tilde{p} = -C_2(g,b,\theta) \tag{4b}$$

Thus, equation (3) implicitly defines industry response to a marketable permit issuance of $\overline{b}$ and equation (4) inplicitly defines industry response to an emission fee.

B. The Regulator's Choice

We now determine the optimal choice of permit issuance, b*, or emission fee, p*. In choosing the optimal permit issuance the regulator wishes to choose b* such that

$$\mathcal{E}[S(g(b^*),b^*,\eta,\theta,\varepsilon)] = \max_{b} \mathcal{E}\,[S(g(b),b,\eta,\theta,\varepsilon)] \tag{5}$$

where $\mathcal{E}$ is the expectation operator over the random variables η, θ, and ε and where $g(b)$ is the reaction function defined implicitly by equation (3). The solution to equation (5) must satisfy first order conditions for a maximum:

$$\mathcal{E}\left\{S_1\frac{dg}{db} + S_2\right\} = \mathcal{E}\left\{[B_1-C_1]\frac{dg}{db} - C_2 - D_1\right\} = 0. \tag{6}$$

But from equations (1) and (3), $B_1 = C_1$; therefore

$$\mathcal{E}\,[-C_2(g(b^*),b^*,\theta)] = \mathcal{E}\,[D_1(b^*,\varepsilon)] \tag{7}$$

Equations (3) and (7) thus implicitly define b*.

Similarly, in choosing the optimal emission fee, we wish to find p* which satisfies:

$$\mathcal{E}\,[S(g(p^*),b(p^*),\eta,\theta,\eta)] = \max_{p} \mathcal{E}\,[S(g(p),b(p),\eta,\theta,\eta)] \tag{8}$$

where once again, $g(p)$ and $b(p)$ are reaction functions and are defined implicitly by equation (4). First order conditions for a solution to (8) are:

$$\mathcal{E}\,[S_1\frac{dg}{dp} + S_2\frac{db}{dp}] = \mathcal{E}\left\{[B_1-C_1]\frac{dg}{dp} - [C_2+D_1]\,\frac{db}{dp}\right\} = 0. \tag{9}$$

But from equations (1) and (4a), we know $B_1 - C_1 = 0$. And from equation (4b), $p = -C_2$. Thus equation (a) can be rewritten as:

$$p^* = \frac{\mathcal{E}\,[D_1(b(p^*),\varepsilon)\frac{db}{dp}]}{\mathcal{E}\,[\frac{db}{dp}]} \tag{10}$$

We can obtain db/dp by totally differentiating equation (4), since (4) holds for all p, not just p*. The result can be solved for db/dp:

$$\frac{db}{dp} = \frac{C_{11}(g,b,\theta) - P_1(g,\eta)}{C_{12}^2(g,b,\theta) + C_{22}(g,b,\theta)[P_1(g,\eta) - (C_{11}(g,b,\theta)]} \tag{11}$$

Thus, equation (4), (10) and (11) implicitly define p*.

III. PRICES VS. QUANTITIES

Our question now is, which optimally designed instrument, p* or b*, gives the greatest expected welfare? Define the relative advantage of prices over quantities as:

$$\Delta \equiv \mathcal{E}\left\{S[g(p^*),b(p^*),\eta,\theta,\varepsilon] - S[g(b^*),b^*,\eta,\theta,\varepsilon]\right\} \tag{12}$$

The quantity Δ defines the difference in total expected social surplus between the two instruments. Clearly, if $\Delta > 0$, emission fees will be preferred to quantities for purposes of regulating emissions; and if $\Delta < 0$, emission permits will be preferred to prices.

In order to evaluate equation (12), we must move from the implicit definition of g(p), b(p) and g(b) in equations (4) and (3), and develop explicit expression for these reaction functions so that the sign of Δ may be deduced. To do this, we will consider second-order Taylor series expansions of B, C and D about the point $(\hat{b},\hat{g}) = (b^*, \mathcal{E}\,[g(b^*)])$:

$$B(g,\eta) \approx \hat{B}(\eta) + \hat{B}_g(\eta)(g-\hat{g}) + \frac{\hat{B}_{gg}}{2} \tag{13a}$$

$$C(g,b,\theta) \approx \hat{C}(\theta) + \hat{C}_g(\theta)(g-\hat{g}) + \hat{C}_b(\theta)(b-\hat{b}) + \frac{\hat{C}_{bb}}{2}(b-\hat{b})^2 \tag{13b}$$

$$+ \frac{\hat{C}_{gg}}{2}(g-\hat{g})^2 + \hat{C}_{gb}(g-\hat{g})(b-\hat{b})$$

$$D(b,\varepsilon) \approx \hat{D}(\varepsilon) + \hat{D}_b(\varepsilon)(b-\hat{b}) + \frac{\hat{D}_{bb}}{2}(b-\hat{b})^2 \tag{13c}$$

Note in the above that the zero and first order terms are random variables, whereas the second order terms are constants. The use of second order approximations and the assumption that the second-order coefficients are constants are probably the strongest assumptions of this paper. These assumptions are the same as those adopted by Weitzman, and the interested reader is referred to the justification and criticism which appeared in the context of the Weitzman paper (see Malcomson, 1978; Weitzman, 1974, 1978; Laffont, 1977). In essence, we are assuming that uncertainty acts to shift the demand, marginal cost and marginal damage functions. The slope of these functions is unaffected by uncertainty.

Differentiating equation (13), one obtains expressions for inverse demand, marginal cost and marginal damage.

$$P(g,\eta) \approx \hat{B}_g(\eta) + \hat{B}_{gg}(g-\hat{g}) \tag{14a}$$

$$C_1(g,b,\theta) \approx \hat{C}_g(\theta) + \hat{C}_{gg}(g-\hat{g}) + \hat{C}_{gb}(b-\hat{b}) \tag{14b}$$

$$C_2(g,b,\theta) \approx \hat{C}_b(\theta) + \hat{C}_{bb}(b-\hat{b}) + \hat{C}_{gb}(g-\hat{g}) \tag{14c}$$

$$D_1(b,\varepsilon) \approx \hat{D}_b(\varepsilon) + \hat{D}_{bb}(b-\hat{b}) \tag{14d}$$

Because benefit, B, is consumer surplus, its slope is the inverse demand for the good, $P(g,\eta)$. Denote the expected value of $\hat{B}_g$, $\hat{C}_g$, $\hat{C}_b$ and $\hat{D}_b$ by $\overline{B}_g$, $\overline{C}_g$, $\overline{C}_b$ and $\overline{D}_b$ respectively. Note first of all, that because we have a competitive market, good price always equals marginal good production cost. Thus, from equation (3), (4a) and (14a,b) for any permit issuance or fee (optimal or nonoptimal), it is always true that

$$\hat{B}_g(\eta) + \hat{B}_{gg}(g-\hat{g}) = \hat{C}_g(\theta) + \hat{C}_{gg}(g-\hat{g}) + \hat{C}_{gb}(b-\hat{b}). \tag{15}$$

In particular, let $b = b^* = \hat{b}$ and take expectations of both sides of equation (15) to obtain

$$\overline{B}_g = \overline{C}_g. \tag{16a}$$

Furthermore, substituting equation (14) into equation (7) yields

$$\overline{C}_b = -\overline{D}_b. \tag{16b}$$

We now use the approximation to benefits, damages, and costs to derive explicit expressions for the reaction functions to p*. Because of our expansion about b* and [g(b*)], we do not need an explicit expression for g(b*). Combining equation (4b) and (14c) and using (15), we obtain, for any p,

$$b(p) = \hat{b} + \frac{[p+\hat{C}_b(\theta)][\hat{B}_{gg}-\hat{C}_{gg}] - \hat{C}_{gb}[B_g(\eta)-C_g(\theta)]}{E} \tag{17a}$$

$$g(p) = \hat{g} + \frac{[\hat{p}+C_b(\theta)]\hat{C}_{gb} + \hat{C}_{bb}[\hat{B}_g(\eta)-C_g(\theta)]}{E} \tag{17b}$$

$$\text{where } E = \hat{C}_{bb}(\hat{C}_{gg}-\hat{B}_{gg}) - \hat{C}_{gb}^{2}. \tag{17c}$$

Because C is strictly convex and demand is not upward sloping, E is always positive. Differentiating equation (17) we obtain

$$b'(p) = \frac{\hat{B}_{gg} - \hat{C}_{gg}}{E} \tag{18a}$$

$$g'(p) = \frac{\hat{C}_{gb}}{E} \tag{18b}$$

neither of which is a random variable. Consequently, the denominator of equation (10) cancels and equation (10), (14d), (16) and (17a) can be combined to obtain

$$p^* F = -\overline{C}_b F \tag{19a}$$

$$\text{where } F = (\hat{C}_{bb}+\hat{D}_{bb})(\hat{C}_{gg}-\hat{B}_{gg}) - C_{gb}^{2}. \tag{19b}$$

From the strict concavity of S in equation (2), we know that F in equation (19a) is negative. Thus, equation (19a) implies

$$p^* = -\overline{C}_b. \tag{20}$$

Equation (17) can now be rewritten as

$$b(p^*) = \hat{b} + \frac{\alpha(\theta)[\hat{B}_{gg}-\hat{C}_{gg}] - \hat{C}_{gb}\beta(\eta,\theta)}{E} \tag{21a}$$

$$g(p^*) = \hat{g} + \frac{\alpha(\theta)\hat{C}_{gb} + \hat{C}_{bb}\beta(\eta,\theta)}{E} \tag{21b}$$

where $\alpha(\theta) = \hat{C}_b(\theta) - \overline{C}_b$ (21c)

$$\beta(\eta,\theta) = \hat{B}_g(\eta) - \hat{C}_g(\theta)$$

Note that $\mathcal{E}(\alpha(\theta)) = 0$. Furthermore, from (15)

$$g(b^*) = \hat{g} + \frac{B(\eta,\theta)}{\hat{C}_{gg} - \hat{B}_{gg}} \tag{21d}$$

We are now ready to evaluate the sign of Δ (equation (12)), the relative advantage of prices over quantities. Substitutiong equation (13) into equation (12), one obtains

$$\begin{aligned}\Delta = \mathcal{E}\{&[\hat{B}_g(\eta)-\hat{C}_g(\theta)]\,(g(p^*)-\hat{g}) - [\hat{C}_b(\theta)+\hat{D}_b(\varepsilon)]\,[b(p^*)-\hat{b}] \\ &+ (\hat{B}_{gg}-\hat{C}_{gg})\,[g(p^*)-\hat{g}]^2/2 - (\hat{C}_{bb}+\hat{D}_{bb})\,[b(p^*)-\hat{b}]^2/2 \\ &- \hat{C}_{gb}[g(p^*)-\hat{g}]\,[b(p^*)-\hat{b}] - [\hat{B}_g(\eta)-\hat{C}_g(\theta)]\,(g(b^*)-\hat{g}) \\ &- (\hat{B}_{gg}-\hat{C}_{gg})\,[g(b^*)-\hat{g}]^2/2\}.\end{aligned} \tag{22}$$

Substituting equation (21) into equation (22) and simplifying reduces equation (22) to

$$\Delta = \left\{\frac{\mathcal{E}\,[(\hat{C}_{gg}-\hat{B}_{gg})\alpha(\theta) + \hat{C}_{gb}\beta(\theta,\eta)]^2}{2[\hat{C}_{bb}(\hat{C}_{gg}-\hat{B}_{gg}) + \hat{C}_{gb}^{\,2}]^2}\right\}\left[\hat{C}_{bb} - \frac{\hat{C}_{gb}^{\,2}}{\hat{C}_{gg}-\hat{B}_{gg}} - \hat{D}_{bb}\right] \tag{23}$$

The denominator of the expression in braces above is E^2 (equation 17c) and, as we have already noted, is never zero. Thus, the braced expression is always nonnegative and the sign of Δ hinges on the sign of the right-hand expression in brackets. Further, a necessary and sufficient condition that there be a preference for one instrument over the other (i.e., $\Delta \neq 0$), is that there be some uncertain in costs or good demand (or both) so that the braced expression is nonzero. Uncertainty in damage is unimportant. This result is embodied in the following proposition.

Proposition: In an industry with a bad (b) produced jointly with a good (g), assume industry costs C(g,b) are strictly convex, that demand for the good is not upward sloping and that damage D(b) is convex. Further assume consumer surplus from the good, production costs and damage are approximated by second order Taylor series expansions with zero and first-order coefficients uncertain. Then the sign of the expected value of the surplus gain (Δ) from using *ex ante* emission fees rather than marketable emission permits for control of b is determined by

$$\text{sign } (\Delta) = \text{sign}\left[\frac{\hat{C}_{gb}^{\,2}}{\hat{B}_{gg} - \hat{C}_{gg}} + \hat{C}_{bb} - \hat{D}_{bb}\right] \tag{24}$$

except when there is neither uncertanty in costs nor good demand (i.e., the numerator of the expression in braces in (23) is zero) in which case $\Delta = 0$.

The result is very similar to that of Weitzman (1974). In fact, the proposition reduces trivially to Weitzman's result for the case of $\hat{C}_{gb} = 0$, no cross-cost effects. With $\hat{C}_{gb} = 0$, the marginal cost of good production is independent of the level of the bad and conversely, the marginal cost of pollution control is idependent of the level of good. In such a case, prices are preferred over quantities if the absolute slope of marginal bads production cost exceeds the slope of marginal damage.

The first two terms on the right-hand-side of (24) constitute the effective slope of the marginal cost of producing the bad, after subtracting out the marginal benefits associated with good production. Thus, as with Weitzman's result, whether or not prices are preferred to quantities depends on the relative slope of marginal damage and marginal (net) costs of bad production.

To see this more closely, define the "market" benefits of producing the bad (excluding pollution damage) as

$$S(b) = B(g,b) - C(g,b) \tag{25a}$$

with g defined by

$$B_g(g,b) = P(g) = C_g(g,b) \tag{25b}$$

The regulator then balances S(b) with D(b) in regulating the output of b. Equation (25a) can be differentiated to obtain (using equation 25b)

$$S''(b) = -\frac{C_{gb}^{2}}{B_{gg} - C_{gg}} - C_{bb} \tag{26}$$

Compare equation (26) with equation (24) and note that in the proposition, prices are preferred if the absolute slope of marginal market benefits ($S''(b)$) exceeds the slope of marginal damage. This is the same result as Weitzman's. It should be noted that equation (25a) cannot be used directly in Weitzman's theorem since $S(b)$ is shared between society and the industry; i.e., some of the benefits acrue to the regulator and some to the firm being regulated.

III. INTERPRETATION

In order to better understand the proposition, we will consider four special cases in more detail: perfectly inelastic and perfectly elastic good demand; and linear and kinked damage (equivalent to perfectly elastic and perfectly inelastic damage). The case of perfectly inelastic good demand is embodied in the following corollary:

Corollary 1: With the assumption of the Proposition, assume further that good demand is perfectly inelastic. If $\hat{C}_{bb} - \hat{D}_{bb}$ is positive (negative), then price (quantity) instruments are the preferred decentralized control instrument, provided there is sufficient uncertainty for there to be a preference ($\Delta \neq 0$).

Proof: In proving this corollary, the limit of the entire expression (23) must be taken as $\hat{B}_{gg} \to -\infty$. The sign of the expression is then equivalent to the sign of $\hat{C}_{bb} - \hat{D}_{bb}$.

This is essentially the same as Weitzman's (1974) result that the performance of fees over standards is determined by the sum of the second derivatives of the cost and damage function. However, in this corollary there may only be uncertainty in good demand and not in costs or damage and still be a preference for one instrument over the other.

These results for the case of inelastic good demand can be seen graphically. Figure 1 focuses on production of bads with both the expected value of marginal costs ($\overline{C}_b$) and damages ($\overline{D}_b$) shown (solid lines) as well as <u>ex post</u> values of these functions (broken lines). The optimal price instrument (p*) and quantity instrument (b*) will be set based on where the expected costs and damages intersect. But the <u>ex post</u> quantities of goods and bads will be determined by the position of the dashed lines. Thus, the shaded areas in the Figure show the welfare losses associated with the two instruments. In the figure we consider two cases distinguished by the sign of $\hat{C}_{bb} - \hat{D}_{bb}$.

In Figure 1a, marginal production costs for the bad are more steeply sloped than marginal damage ($\hat{C}_{bb} - \hat{D}_{bb}$ is positive), which from equation (24) should imply the superiority of emission fees. This is, in fact, illustrated, because the shaded welfare loss associated with fees in Figure 1a appears to be considerably smaller than that associated with quantity controls. Exactly the opposite effect occurs in Figure 1b, where $\hat{C}_{bb} - \hat{D}_{bb}$ is assumed negative.

The opposite of inelastic demand is the case of a totally elastic demand function for the good.

Corollary 2: With the assumptions of the proposition, and assuming that demand for the good is totally elastic, if

$$\frac{\hat{C}_{gg}\hat{C}_{bb} - \hat{C}_{gb}^{2}}{\hat{C}_{gg}} - D_{bb}$$

is positive (negative), then price (quantity) controls are the preferred decentralized control instruments, provided there is sufficient uncertainty for there to be a preference ($\Delta \neq 0$).

Proof: Because inelastic good demand implies $\hat{B}_{gg} = 0$, the corollary follows directly from the Proposition.

This corollary is similar to the one encountered in the case of inelastic good demand. The difference is that instead of the slope of the marginal cost curve for the bad (C_{bb}), we deal with the ratio of the determinant of the Jacobian of the cost function to the slope of the marginal cost curve for the good--in a sense the curvature of the overall cost function relative to the curvature of the cost of good production. Refer to Kolstad (1982) for a detailed graphical interpretation of this corollary.

Finally, we consider the slope of the pollution damage function.

Corollary 3: With the assumptions of the proposition, and assuming there is sufficient uncertainty ($\Delta \neq 0$), then for linear damage (kinked damage), a price (quantity) instrument is preferred for pollution control.

Proof: Linear damage implies $\hat{D}_{bb} = 0$, and kinked damage implies $\hat{D}_{bb} \to \infty$. The corollary follows directly from the Proposition.

This corollary, too, is in the spirit of Weitzman, in that constant marginal damages are best conveyed to the firm through an emission fee. Conversely, if damage is kinked, then marginal damage is changing rapidly in the vicinity of the kink so a quantity message is best relayed to the firm. Further discussion of these corollaries may be found in Kolstad (1982).

V. CONCLUSIONS

The basic purpose of this paper has been to extend Weitzman's (1974) analysis to the case where pollution is produced jointly with a good and there is uncertainty in good demand as well as production costs and pollution damage. This would appear to be a much more realistic situation, closer to that found in the "real" world. Our basic result is that the relative slope of damage and pollution costs still determine the preference for price or quantity instruments. However, uncertainty in good demand alone is adequate to yield a preference for one over the other; there need not be uncertainty in costs.

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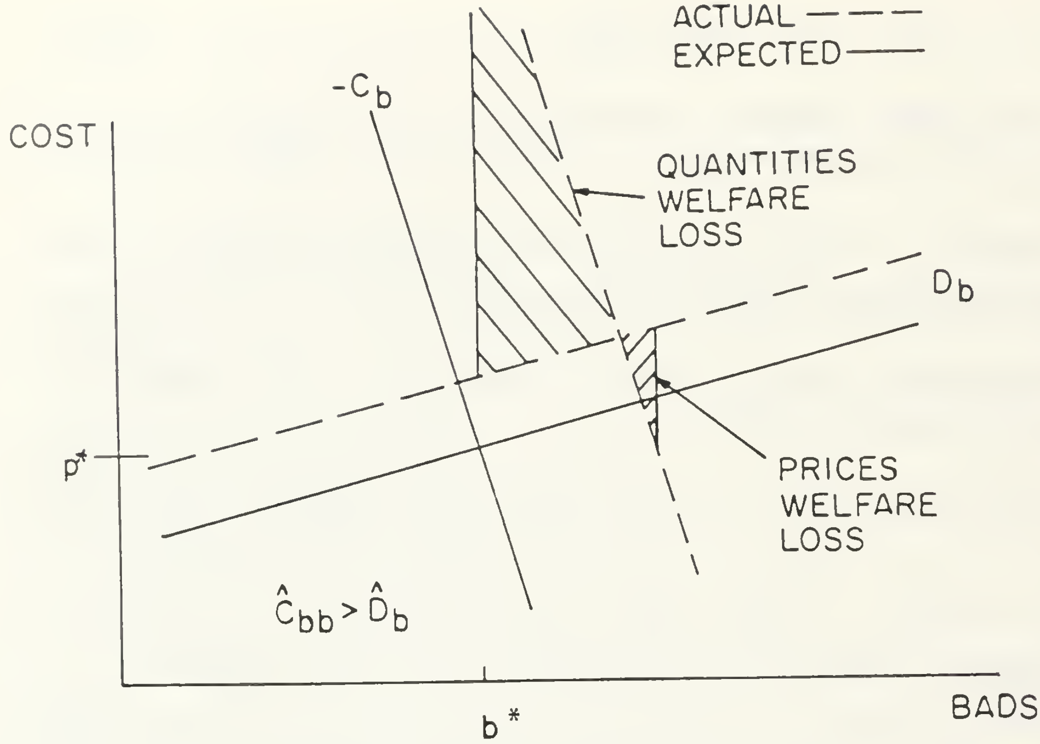
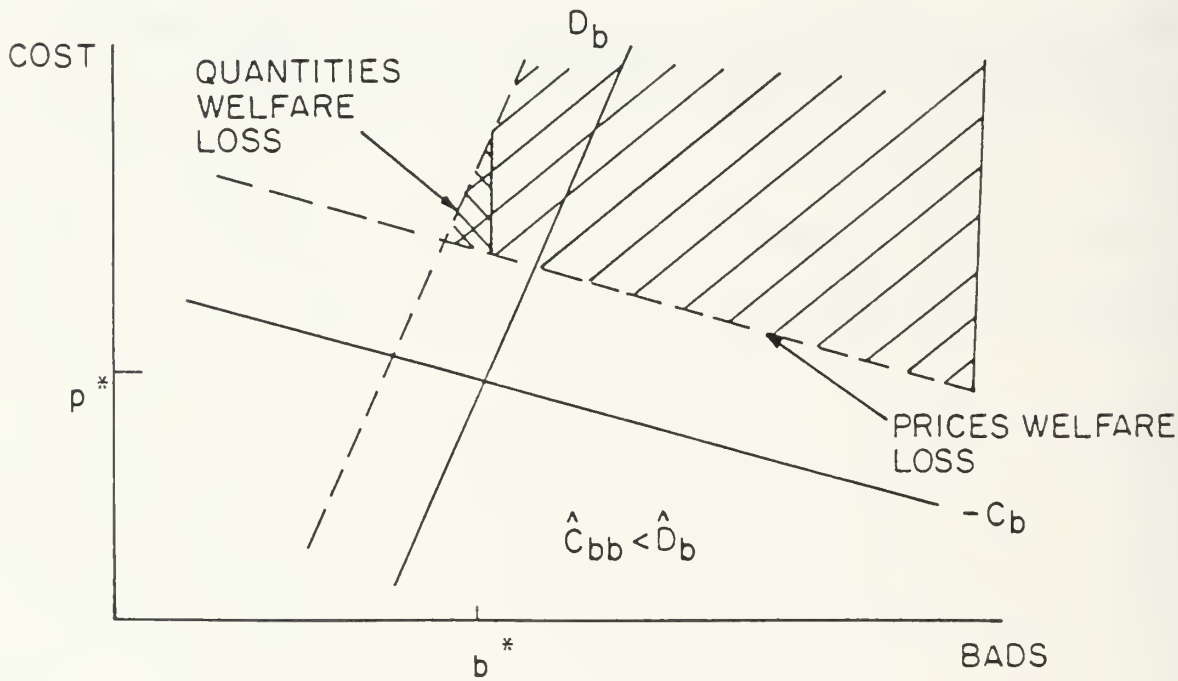


(a)



(b)

Figure 1: Inelastic good demand.